PROPOSAL ABSTRACT:

Title. Patrolling strategies to avoid fare evasion in public transportation systems: a Stackelberg game approach.

Abstract. Many public transportation systems in major urban areas, such as the German subway networks, Brussels tram system, and the London, Rome, and Santiago (Chile) bus systems, require passengers to purchase tickets before entering. Yet, they are not physically forced to do so because physical ticket barriers are not always possible, cost-efficient, or desirable. Therefore, to inhibit fare evasion, patrol units move throughout the transit system to inspect passengers. The objective of the transit operator, which schedules the inspection paths that each patrol follow during their working day, is to catch the maximum number of fare evaders (and to collect the maximum amount of fines). At the same time, opportunistic passengers have to choose between buying a ticket or taking the risk of paying the fine if they are caught.

The problem can be seen as a Stackelberg game in which the transit operator (*the leader*) has to establish a mixed (randomized) patrolling strategy, and the opportunistic passengers (*the followers*), observing this strategy, decide to buy a ticket or not, maximizing their cost (depending on the probability of being inspected). As far as is known, only two heuristics to return a feasible mixed patrolling strategy have been developed by Yin et al. (2012) and Jiang et al. (2013), which lack a guarantee in terms of the optimality gap, i.e., the quality of the solution resulting from these heuristics is unknown. Furthermore, these heuristics are narrow in terms of the assumptions used to model the behavior of followers, because they consider that opportunistic passengers are rational, have complete information on the distribution of inspection probabilities, and take fixed paths to reach their destination. On the other hand, the heuristics developed are also narrow in the inspection policies used by transit operators.

This project aims to address the problem of scheduling random patrols to inhibit fare evasion in public transportation systems, from the Stackelberg game point of view, taking into account realistic models for the reaction of opportunistic passengers, realistic models for the actions of the transit operator, and guaranteeing the quality of the solutions in terms of the optimality gap. Thus, the expected result of the project is practical and novel mathematical tools to schedule random fare inspections, in public transportation systems, to inhibit the fare evasion of the most significant number of opportunistic passengers.

The problem of scheduling random patrols to inhibit fare evasion in public transportation systems can be formulated as a bi-level optimization problem. At the first level, the transit operator determines the set of patrolling paths and their respective probabilities of being selected (mixed patrolling strategy) that contribute the most to their objective. Then, at the second level, opportunistic passengers respond by optimizing their objective function (depending on the behavior of the opportunistic passengers), given the probability of being fined. This approach assumes that the set of all patrolling paths is known, which is impossible to know in real transportation networks because this set grows exponentially with the size of the transportation network. In this project, it is proposed to explore decomposition methods to address the combinatorial nature of this problem. An alternative is to use a column generation approach. However, it is known that this approach has stability problems that affect computational times. A second alternative to be explored in this project is the use of the Bienstock-Zuckerberg Algorithm, which has proven to be an excellent alternative to the column generation approach in several scheduling problems. A third alternative, similar to the method developed by Yin et al. (2012) and Jiang et al. (2013), is to return a set of patrolling paths and their respective probabilities of being selected from the inspection probabilities set by the transit operator. This approach, where inspection probabilities are explicitly determined, defines a *marginal patrolling strategy*, which is operationally useful for the transit operator as long as a mixed patrolling strategy is returned. In this project, it is proposed to explore the relationship between the marginal and mixed patrolling strategies, and determine their equivalence in terms of optimization problems because if they are not equivalent, then a mixed patrolling strategy returned from an optimal marginal patrolling strategy, is not optimal.

In this project a three-year work plan is proposed, where the workload has been carefully balanced to achieve the proposed objectives.